

Second Semester B.E./B.Tech. Degree Examination, June/July 2026 Multivariable Calculus and Numerical Methods : ME Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks, L: Bloom's level, C: Course outcomes.
3. VTU Formula Hand Book is permitted.*

Module – 1			M	L	C
Q.1	a.	Evaluate the integral $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$	7	L3	CO1
	b.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy \, dy \, dx$ by changing the order of the integration.	7	L3	CO1
	c.	Prove the Relationship between Beta and Gamma functions : $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$	6	L3	CO1
OR					
Q.2	a.	Evaluate $\iint xy(x+y) \, dy \, dx$ taken over the area between $y = x^2$ to $y = x$	7	L3	CO1
	b.	Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4, z = 0$	7	L3	CO1
	c.	Show that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\frac{\pi}{2}} \sqrt{\sin\theta} \, d\theta = \pi$	6	L3	CO1
Module – 2					
Q.3	a.	Solve the D.E. $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$	6	L3	CO1
	b.	Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 3^x$	7	L3	CO1
	c.	Solve $x^2y'' - 3xy' + 4y = (1+x)^2$	7	L3	CO1
OR					
Q.4	a.	Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$	6	L3	CO1
	b.	Solve: $y'' + 3y' + 2y = 12x^2$	7	L3	CO1
	c.	Solve $(2x+1)^2y'' - 6(2x+1)y' + 16y = 8(2x+1)^2$	7	L3	CO1
1 of 3					

Module – 3

Q.5	a.	If $\vec{F} = xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$, find the $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ at $(2, -1, 1)$.	6	L3	CO1
	b.	Find the directional derivative of the function $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ along $\hat{i} + 2\hat{j} + 3\hat{k}$	7	L3	CO1
	c.	Find the angle between the surfaces or normal surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.	7	L3	CO1

OR

Q.6	a.	P.T $\vec{F} = (y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$ is irrotational. Also find a scalar point function ϕ such that $\vec{F} = \nabla\phi$.	7	L3	CO1
	b.	If $\vec{F} = (3x^2 + 6y)\hat{i} - (14yz)\hat{j} + (20xz^2)\hat{k}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ from the point $(0, 0, 0)$ to $(1, 1, 1)$ along the curve given by $x = t$, $y = t^2$, $z = t^3$.	7	L3	CO1
	c.	Evaluate $\int_C (xy - x^2)dx + x^2ydy$ where C is the closed curve bounded by $y = 0$, $x = 1$ and $y = x$.	6	L3	CO1

Module – 4

Q.7	a.	Find the real root of the equation $x \log_{10} x = 1.2$ by Regula falsi method, Carry out 3 iterations.	6	L3	CO2												
	b.	The population of a town is given by the following data <table border="1"> <tr> <td>Year</td><td>1971</td><td>1981</td><td>1991</td><td>2001</td><td>2011</td></tr> <tr> <td>Population</td><td>19.96</td><td>39.65</td><td>58.81</td><td>77.18</td><td>94.58</td></tr> </table> Using appropriate interpolation formula calculate the increase in the population from the year 1975 to 2005.	Year	1971	1981	1991	2001	2011	Population	19.96	39.65	58.81	77.18	94.58	7	L3	CO2
Year	1971	1981	1991	2001	2011												
Population	19.96	39.65	58.81	77.18	94.58												
	c.	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{1^{rd}}{3}$ rule by taking 5 ordinates (4 equal parts) and hence deduce an approximate value of π .	7	L3	CO2												

OR

Q.8	a.	Use Newton Raphson method to find the real root of the equation $xe^x = 2$, correct to 3 decimal places.	7	L3	CO2												
	b.	Find $f(0.05)$ using Newton divided difference formula given that <table border="1"> <tr> <td>x</td><td>0</td><td>2</td><td>3</td><td>5</td><td>6</td></tr> <tr> <td>f(x)</td><td>0</td><td>6</td><td>21</td><td>105</td><td>186</td></tr> </table>	x	0	2	3	5	6	f(x)	0	6	21	105	186	7	L3	CO2
x	0	2	3	5	6												
f(x)	0	6	21	105	186												
	c.	Evaluate $\int_4^{5.2} \log_e x \, dx$ using Simpson's $\frac{3^{th}}{8}$ rule by taking 6 equal parts.	6	L3	CO2												

Module – 5

Q.9	a.	Use Milne's predictor –corrector method Given the differential equation, given $\frac{dy}{dx} = x^2(1 + y)$, find y at x=1.4 . Carry out two corrections for the solution, given that y (1) =1, y (1.1) =1.233, y (1.2) =1.548, y (1.3) =1.979.	7	L3	CO2
	b.	Using the modified Euler's method, solve the IVP $\frac{dy}{dx} = x + y^2, y(0) = 1$ at points x=0.1 in steps of length h=0.1. Carry out two modifications.	6	L3	CO2
	c.	Using Runge Kutta method of fourth order solve $\frac{dy}{dx} = \frac{(y^2 - x^2)}{(y^2 + x^2)}$ for x = 0.2 given that y = 1 at x = 0 initially , by applying Runge – Kutta method of order 4.	7	L3	CO2
OR					
Q.10	a.	Using the Taylor's series method , find an approximate solution correct to four decimals at x=0.1 for the IVP $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$	6	L3	CO2
	b.	Using RK method of fourth order , to find y(0.1), given that $\frac{dy}{dx} = x^2 - y$ and y(0) = 1 take h = 0.1	7	L3	CO2
	c.	Use Adams -Bash forth method to find the value of y at x=0.8, given $\frac{dy}{dx} = x - y^2$, where y(0)=0 , y(0.2)=0.02 , y(0.4)=0.0795 , y(0.6)=0.1762. Apply corrector formula twice.	7	L3	CO2

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